Discourse Moves for Mathematical Discussion of Generalization: Unit 4, Production Processes
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In previous units, students invented statistics and their inventions helped them understand the nature of the problems that are solved by statistical conventions, such as median and IQR. In this unit, statistics are extended to a new context of production (e.g., toothpick factory), where statistics of central tendency estimate the target value of the process. Statistics of variability are interpreted as indicating the consistency of the product produced. For different methods of production, choose a few different kinds of methods for center and for variability, and engage students in a conversation about whether or not the methods of production make a difference for attaining the target value or for producing consistent products.

Eliciting a method: *Ask several students to say what information the measure of center or precision tells about the production process. Alternate between questions in Box 1 and Box 2 to build relations between the statistics and characteristics of different production methods that result in either change or stability in the statistic across differing methods of production.*

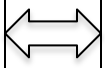
Box 1: Interpreting the method

"Take a look at this display—a combination of all of our data. What do you notice? What does the display tell us about the (_____ process)?"

"_____, what does the (mean, median) tell us about these data?"

"_____, what does the (IQR, average deviation, range) tell us these data?"

Separate the displays by type of process.
Ask: " Take a look at these two displays. What do you notice?"
"What does the (statistic) tell us about these two different ways of producing (toothpicks, candies, walking rates)?"



Box 2: Explaining stability and change in statistics across data sets

"_____, why did (statistic) change/not change by much?" What about the process made that happen?

"_____, why do you think this statistic gives us a similar estimate of the target value in each of these processes?"

"Did the (statistic) change like you had expected from one process to the next? Why do you suppose it did/did not?"

Building Collective Understanding

1. Restate that student's hypothesis or have someone else restate the hypothesis to make the hypothesis public (yes-anding/making it public)

- "_____ , can you restate what _____'s argument was about what this (statistic) tells us about the process?"
- "_____, tell us what _____ thinks changed about the (measure of center, or measure of variability) between these two data sets."

2. Ask any extension/clarification questions if necessary to help others understand. Make sure to clarify whether students are referring to <u>methods of center/variability</u> or an aspect of the <u>production processes</u>.

- "What do you mean when you say _____?"
- "I'm not sure I understand what you mean by _____."
- "When you say "better," are you referring to the method or the production process?"
- "So the *<u>range</u>* is less accurate or the *<u>picket fence method</u>* is less accurate?"

Eliciting a response to the hypothesis:

Ask the author if that is what was intended, and/or open it up to other students.

- "_____, do you agree with how _____ said that the <u>*median*</u> predicts the target value? "
- "_____, do you agree with how _____ said that the <u>*IQR*</u> predicts how consistently toothpicks were packaged (or how consistently people walked at a particular rate)?"
- "Do you agree that the range changed because it was harder to reach a reliable target value?"
- "Do you agree that we get a similar target value with the median because we have about the same number of overestimates and underestimates, even if they are more or less spread apart?"

Connective statements/questions:
1. *Ask questions or make comments to get students to think about similarities/differences between methods and which measures would be more informative with particular data qualities.*

- "Which statistics would give you similar values across different methods?"
- "What about the methods led to these (differences, similarities) in the value of this (statistic)?"
- "Which statistic is the most helpful for (predicting the target value, telling us about the consistency of the process, being sensitive to change in the method of packaging, etc.)?"
- "Which statistic would you trust less when trying to find the (target value, consistency of the process, etc.)?"

Transformation:
2. *Ask students to apply their reasoning to imagined data points or data sets. The purpose of these questions is to create situations that will* ***very*** *clearly show why certain reasoning is problematic when it is generalized to other data sets. Therefore, this requires quick thinking about how the reasoning is problematic* ***and*** *what kind of situation would highlight that problem clearly.*

- "If the toothpick factory packaging resulted in a few counts like this (create a very large or small count), how would it affect the measures of center? Of consistency?" [Ask analogous question for rate walks)
- "What would happen to this measure if the entire school tried out the two production processes/did rate walks?"
- "What if we used this method on a set of data that looks like this?" (Draw imagined data set)

Identifying, creating, and pressing on contradictions:
Contradictions help prevent students from over-generalizing and offer ways to connect ideas. By pointing out ways that the implications of different methods contradict, students will have to either revise their methods or evaluate the utility of each method to reconcile the contradiction.

- "_____ stated that the mean will always give us the best guess of the target value. Can anyone think of a data set the mean might not work on as well?"
- "_____ stated that the average deviation will always give us a good estimate of the consistency of (our packing of toothpicks or our rate of walking). Can anyone think of a data set where the average deviation might not work as well?"
- "_____ said the range is always a good measure of precision, but what about a data set in which there is a value that is really far from the center clump? Would the range give us a good measure of precision there? Why/why not?"

Pulling it together:
Make a brief summary statement with a "big idea" that students have come to through discussion. Think of it as a restatement, but you may want to add something extra to help make this idea salient. Include both points of consensus as well as issues to remain "on the table" for further discussion. Record on anchor chart.

- "I think the points we're agreeing on here are <u>*(points of consensus).*</u> We haven't yet come to agreement on <u>*(items that will remain "on the table")*</u>."
- "What I think I hear people saying is _____. We'll continue our discussion about _____ next time."