Investigating Chance Unit 5

Buttoned Shirts



A machine that puts buttons on dress shirts sometimes adds an extra button or leaves off a button. The company collected data about **20** shirts the machine made in an hour and **counted how often a button was missing or an extra button was added.**

Here is the data they collected:

	Missing a Button	Correct	Extra Button Added
Shirt Count	4	12	4

1. Given this data, what is the probability a shirt will be **missing** a button?

Show how you calculated it.

2. Given this data, what is the probability a shirt will have an **extra** button?

Show how you calculated it.

3. Given this data, what is the probability the shirt will have the **correct** number of buttons?

Show how you calculated it.

Investigating Chance Unit 5

4. Given the data, **label the spinner** on the right with the **probabilities** that the shirts will be missing a button, have an extra button, or have the correct number of buttons.

Label the spinner by writing: <u>M</u> for missing buttons,

 $\overline{\mathbf{E}}$ for extra buttons, and

 $\underline{\underline{C}}$ for the correct number of buttons



Carnival 1



At the carnival, there is a slot machine. When you pull the handle, the numbers 1, 2, and 3 spin past both windows until the machine stops. Every time it stops, each window has an equal chance of showing the numbers 1, 2 or 3. Players win prizes depending on the numbers that show up in the two windows

1. One possible result of playing the game is shown above. The player got a 3 in the left window and 1 in the right. List all the other possible results

Answer: (3,1),

2. A player wins a special prize if the sum of the two numbers is 5 or higher.

What are the possible ways to get a **sum of 5**?

What are the possible ways to get a **sum of 6**?

What is the **probability of getting a sum of 5 or higher** and winning the special prize? Explain how you got your answer.

Investigating Chance Unit 5

Fair Coin

Part 1

John flips a coin 10 times. He finds that 90% of the coin tosses are heads.

- a) How many heads did he get?
- b) How many tails did he get?
- c) If this were all the information that John had, would he tend to believe that the coin was fair? Explain why.

Part 2

John wants to know if the coin is fair. A fair coin will land on heads 50% of the time and tails 50% of the time. During several days, he collects 400 samples of 10 coin flips and for each sample writes down the percent heads. After looking at the results he decides the coin really is fair.

- a) Draw below what you think the shape of the 400 samples of percent-heads data looks like.
- b) Label the lowest percent-heads and the highest percent-heads that are likely to be in the samples.
- c) Label the median value.

Percent Heads, 400 samples, 10 coin flips per sample

d) Explain your thinking for (a)-(c)

Part 3

If John flipped the coin 50 times for each of the 400 samples, instead of 10 times:

- (a) Draw below what you think the shape of the 400 percent heads data looks like for the sample size of 50.
- (b) Label the lowest percent heads and the highest percent heads that are likely.
- (c) Label the median value.

Percent Heads, 400 samples, 50 coin flips per sample

(d) Explain your choices for (a)-(c).

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Unit Quiz Scoring Guide: Buttoned Shirts

Buttoned Shirts

Questions	1-3: Probability shirts will have a r	nissing/extra/correct number of buttons
Level	Performance	Example
Cha(3c+)	 Correctly quantifies probability as the ratio of the number of target outcomes to all possible outcomes by providing the correct percentages or ratios for "Missing", "Extra" and "Correct." Implies understanding of the notion of probability as a ratio based on an explanation or through clear calculations. (Students that provide very clear explanation and/or correct calculations but presents errors in mathematical notation can be included in this level if it is considered that those errors do not alter the interpretation of probability.) 	Show how you calculated it. $\frac{4}{20} = \frac{124444}{20} = \frac{20}{20}$ Show how you calculated it. $\frac{4}{20} = \frac{1}{20}$ Show how you calculated it. $\frac{4}{20} = \frac{1}{20}$ Show how you calculated it. $\frac{4}{20} = \frac{1}{5} = \frac{2}{5} = \frac{29}{20} = \frac{2076}{120}$ Example with error in mathematical notation: $\frac{4}{20} = \frac{1}{5} = \frac{1}{5} = \frac{29}{20} = \frac{2076}{120}$ Example with error in mathematical notation: $\frac{4}{20} = \frac{1}{5} = \frac{1}{5$
Cha(3c)	Correctly quantifies probability as the ratio of the number of target outcomes to all possible outcomes by providing the correct percentages or ratios for "Missing", "Extra" and "Correct" but no explanation or calculations are provided.	Hout of 20

Unit Quiz Scoring Guide: Buttoned Shirts

Investigating Chance Unit 5

Cha(2b)	The response indicates that the student understands that the probability is a relationship between the frequency and a total, but the student is not able to correctly express the relationship by failing to correctly identify the numerator OR the denominator (the frequency). Student may use a consistent but incorrect value to represent the total. If a student	Show how you calculated it.
	makes mistakes both in the numerator AND the denominator he/she should be scored as NL(ii).	
Cha(2b-)	The response indicates that the student understands that the probability is a relationship between the frequency and a total, but the student expresses the inverse relation, using the frequency as denominator. The student in this level may or may not have correctly identified the total (the numerator).	Show how you calculated butten Show how you calculated
Cha(1b)	Provides the frequencies rather than ratios. All the frequencies are correct. A student in this level may present errors in mathematical notation, such as the inclusion of a '%' sign, but the numbers accurately reflect the frequencies.	$\frac{12}{\text{Example with error in mathematical notation:}}$
NL(ii)	Relevant but incorrect response	 "M=5 shirts, E=5 shirts, C=10 shirts" "M=36, E=21, C=37"
NL(i)	Response is irrelevant, unclear, or a restatement of given information.	 "M=extra, E=correct, C=missing" "I don't know."*
М	Missing response	

Unit Quiz Scoring Guide: Buttoned Shirts

Investigating Chance Unit 5

Question 4 Buttoned S	: Label the spinner, missing Shirts and Modeling Variabi	g/extra/correct number of buttons lity (MoV)
Level	Performance	Example
MoV(3a)	Use a chance device to represent a source of variability or the total variability of the system.	
	• May use letters, shading, patterns or arrows to label sections on the spinner. Labels	MM/r M
	spinner. Labers sections on the spinner to represent proportions consistent with the data provided.	$\begin{array}{c c} C & \overline{E} \\ \hline C & C \\ \hline C & C \\ \hline C & C \\ \hline \end{array} \end{array}$
	~	·
NL(ii)	Show some ideas in correct direction (i.e., vaguely relevant attempt). May recognize the relation between relative proportion of passible outcomes and	
	their likelihood (shows evidence of part-whole relations) but makes an error on the graphical display.	me c c c c c c c c c c c c c c c c c c c
NL(i)	Response is irrelevant, unclear, or a restatement of given information.	*
М	Missing response	

Carnival

Question	estion 1: List other possible results		
Level	Performance	Example	
Cha(5c)	Student describes a sample space by listing ALL permutations of outcomes.	 Example 1: Answer: (3,1) (3,2) (3,3) (2,2) (3,2) (1,3) (1,1) Example 2: (3,1).(3,2).(1,2),(1,3),(2,3).(2,1) (3,3) (2,2), (1,1) Note: (3, 1) is already listed, so students do not need to repeat it. However, students should list (1, 3) to get Cha5(c). 	
Cha(5c-)	Student describes a sample space by listing at least three possible permutations of outcomes, but missing others.	 Example 1: Student lists all but one possible permutation. 13, 31, 11, B3, 32, 1221, 24 Example 2: Student lists most of the permutations. 3 a special prize if the sum of the two numbers is 5 or higher. Example 3: Student lists some possible permutations. 3 12 2 13 Note: (3,1) is already listed, so students do not need to repeat it. 	
Cha(5b)	Student describes a sample space by listing ALL possible combinations of outcomes, but does not consider permutations.	 Example: [3,1] (3,2] (3,3) [1,1] (1,2] (2,2) Note: (3, 1) is already listed, so students do not need to repeat it. 	

Investigating Chance Unit 5

Cha(5b-) Cha(5a) NL(ii)	Student describes a sample space by listing at least three but not all possible combinations of outcomes. Student constructs a new variable by joining outcomes of events (i.e., create compound/aggregate outcomes). The student lists one or two sums (or pairs). Response is relevant in that a) student gave values that came from the slot machine, but the responses do not show order or combination or b) the student includes additional responses that could not come from the slot machine.	 (2,3), (1,2), (3,1) * Note: (3, 1) is already listed, so students do not need to repeat it. Example 1: Student lists one permutation of the example. I J JR Example 2: Student lists 2 pairs that are permutations of one another. (1,2) and (2,1) Example 1: I I I I I I I I I I I I I I I I I I I
		31,13, 1,3,0
NL(i)	Response is irrelevant; no values are provided for the slots.	 Example 1: Here is 14 What? Answer: (3,1), Example 2: They bosth spin Selest. They odd numbers, Mey bosth spin Selest. They odd numbers,
М	Missing response	

Question 2a: Possible ways to get a sum of 5		
Level	Performance	Example
Cha(5c)	Describe a sample space by listing <i>all</i> possible permutations of outcomes. • The student lists (2, 3) AND (3, 2).	• Example: 23,32,
Cha(5b)	Student describes a sample space by listing all possible combinations of outcomes, but does not consider permutations. The student lists either (3, 2) OR (2,3).	• Example: 3+2=5
NL(ii)	Response is relevant in that student gives spinner values, but are incorrect: Student either uses numbers from the spinners but summed to incorrect values, or used additional numbers not on the spinner. Student may also list probabilities instead of spinner values. Additionally, responses that talk about the "number of possible ways" are considered relevant.	• Example 1: $ \lambda H + \lambda H $
NL(i)	Response is irrelevant; no values are provided for each spinner.	 Example 1: 5/right 0/rong Example 2: 5/55

Investigating Chance Unit 5

		• Example 3:
		Ho -2 - 17 - 70 - 70 - 70 - 70 - 70 - 70 - 70
М	Missing response	

Question 2c: Probability of getting sum 5 or higher		
Level	Performance	Example
Cha(6a)	For compound (aggregate) events, relate sample space to probability.	three combows get sor higher and There are of total combows
	• If a student gets a probability of 1/3 (or equivalent), he/she is scored a 6(a) unless there is evidence showing that the student found that probability in a way that did not coordinate the complete sample space to the partial sample space. If the student said the probability was 1/3 and did not show any work, then he/she may be scored a 6(a).	9
Cha(6a-)	 For compound (aggregate) events, relate sample space to probability. Students at 6a- either a) did not 	• Example 1: This student had 6 total outcomes and two that were equal to 5. Like other students, he assumed 5 or more meant sums equal to 5. Because of his explanation, we can assume that he is coordinating to the sample space.

Cha(2h)	count all possible ways of getting 5 or higher (e.g. only counted the ways of getting 5) and/or b) the total number of sums is off. The students' response might equal 1/3 (the correct probability) but be obtained in a different way. Students who consider only combinations would actually calculate the probability as 2/3.	 Hookid at the watt you cangets of higher and how many number combinations there are. Example 2: Student had found 6 total outcomes for the sample space and he had noted 2 ways of getting 5 and 1 way of getting 6 so he only found probability for a sum of 5 rather than a sum of 5 or over. 2046 Example 3: the special prize? Explain how you got your answer. The probability of getting a sum of 5 or higher is 3/7 becomes there are are a numbers and 3 are higher than 5 or higher.
Cha(3b)	Recognize the relation between relative proportions of possible outcomes and their likelihood.	• Example: c. What is the probability of getting a sum of 5 or higher and winning the special prize? Explain how you got your answer. Not 11/11/19 because there is not very many Ways to get 5 or a higher H.
NL(ii)	Students' responses are related to the question, but do not relate the outcomes to probability. For instance, a student might provide incorrect probabilities that are not consistent with the outcome space they listed for question 1. Students who obtained the correct probability of 1/3 but not by	 Example 1: I think that the probibility of getting a som of 5 or higher 1 = 2 differ whys. Example 2: The probability of getting a sum of 5 is 3+250r 2+2+1=5 Example 3:

Investigating Chance Unit 5

	sample space to probability are also considered NL(ii).	(2,3)(3,2)(3,3) (2,3)(3,2)(3,3) (2,3)(3,2)(3,3) (3+3=6+he) higher numbers than \mathcal{B}_{s}
		• Example 4: 1 Oct of 3 Begause there are three players
		• Example 5: This student found the correct probability, but by a method not using his sample space. His explanation is shown on the left and the sample space he wrote in Part 1 is shown on the right.
		1 out of 3 because 3/2 2 2 2 2 3 2 4 2 5 2 5 2 5 2 5 2 6 3 12 2 3 1 12 2 12 2 12 2 12 2 12 2 12 2 12 2 12 2 12 2 12 2 12 2 12 2 13 1 14 2 15 2 16 3
NL(i)	Response not related to the problem. For instance, a student provides values that are not probabilities, e.g., greater than 1, or less than 0.	 Example 1: I got my anar by abbing 5 figures and I got 10 and so if you get one the four more and if you get tow more you could wern. Example 2:
		• Example 3:
		I didn't get the question?
М	Missing response	

Unit Quiz Scoring Guide: Fair Coin

Fair Coin

Part 1: H	Part 1: How many heads?		
Level	Performance	Example	
Cha (4e)	Student relates 90% to 9 heads and suggests that the coin may not be fair.	• "It's 9 heads, 1 tail. If I don't have any other information, then I might guess that the coin is not fair because if it were, I would expect 5 heads."	
		• "It's 9 heads and 1 tail, but it still might be fair. I will need to see more flips."	
Cha (3c)		• "It's 9 heads and 1 tail. I think the coin is fair" (or not fair, but does not cite expected frequency).	
NL		No relation between percent and frequency	

Part 2: Shape of the Sampling Distribution, n =10			
Level	Performance	Example	
CoS(4d)	Student suggests mdn of 50% and a bell shaped distribution with some, but fewer, extreme values. Justifies range by sample size.	 "This shape (approximate bell) because with this small sample, you get some 10%, 90%, but most are around 50% - that's where the mdn will be, because the coin is fair." 	
CoS(4c)	Student suggests mdn of 50% and a bell shaped distribution with some, but fewer, extreme values.	• "This shape (approximate bell) because you get some 10%, 90%, but most are around 50% -that's where the mdn will be, if the coin is fair."	
CoS(4b)	Student indicates awareness of variability in sample due to chance.	"Not sure about the shape but they won't all be the same, just by chance. But they should be around 50%."	
CoS(4a)	Student recognizes	• "Not sure about the shape but it won't	

Unit Quiz Scoring Guide: Fair Coin

	variability.	always be the same. It changes."
NL	Relevant response.	"I don't know, you can't tell."

Part 3: Shape of the Sampling Distribution, n=50			
Level	Performance	Example	
Level	Performance	Example	
CoS 4D	Student suggests mdn of 50% and a bell shaped distribution with less variability than for n=10. Justifies less variability by increase in sample size.	• "This shape (approximate bell, with less variability than previous sample) because when you (have) larger samples, it does not change as much from sample to sample. Most are around 50% -that's where the mdn will be, because the coin is fair."	
CoS 4C	Student suggests mdn of 50% and a bell shaped distribution with less variability. Does not mention sample size.	• "This shape (approximate bell) because now you (there are) more around 50% - that's where the mdn will be, if the coin is fair."	
CoS 4B	Student indicates awareness of variability in sample due to chance.	• "Not sure about the shape but they won't all be the same, just by chance. But they should be around 50%."	
CoS 4A	Student indicates awareness of variability in sample.	• "Not sure about the shape but it won't always be the same. It changes."	
NL	Student attempts solution.	• "I don't know."	